

Final Exam

Name \_\_\_\_\_

Math 397 Spring 2005

Signature \_\_\_\_\_

Circle your instructor's name: Leuschke Graves Latour Meyer

**READ THIS BEFORE YOU BEGIN**

This examination contains 11 problems on 11 pages. Point values are indicated with a total of 200 points. It is your responsibility to make sure that all problems and pages are present. You may use a non-symbolic graphing calculator. No books or notes are allowed on this exam. Your solutions must be written legibly and contain all of the necessary steps which enabled you to arrive at your answer to receive full credit for the problem. Unsupported answers will receive little or no credit. If there is a space for your answer, write your answer in the space. Otherwise circle your final answer.

**DO NOT WRITE IN THE TABLE BELOW**

<b>Problem</b>	<b>possible</b>	<b>actual</b>
1	25	
2	15	
3	20	
4	15	
5	20	
6	20	
7	25	
8	20	
9	10	
10	15	
11	15	
<b>Total</b>	<b>200</b>	

1. Let  $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$  and  $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ . Find

(i)  $\mathbf{a} + \mathbf{b}$ ;

(ii) The unit vector in the direction of  $\mathbf{b}$ ;

(iii)  $\mathbf{a} \cdot \mathbf{b}$ ;

(iv)  $\mathbf{b} \times \mathbf{a}$ .

(v) Are  $\mathbf{a}$  and  $\mathbf{b}$  orthogonal to each other? Explain briefly.

i) \_\_\_\_\_

ii) \_\_\_\_\_

iii) \_\_\_\_\_

iv) \_\_\_\_\_

v) \_\_\_\_\_

2. Find an equation in standard form (i.e.  $Ax + By + Cz = D$ ) for the plane containing the points  $P(5, 2, -1)$ ,  $Q(2, 7, 2)$  and  $R(4, 5, 1)$ .

2) \_\_\_\_\_

3. Let  $\mathbf{r}(t) = t\mathbf{i} - \frac{2}{t}\mathbf{j} - t^2\mathbf{k}$ .

i) Give a set of parametric equations for the line tangent to the curve  $\mathbf{r}(t)$  at  $t = 1$ .

i) \_\_\_\_\_

ii) Find  $\mathbf{T}(t)$ , the unit tangent vector for  $\mathbf{r}(t)$ .

ii) \_\_\_\_\_

4. For each part, find the limit if it exists or *show* why the limit doesn't exist.

i)

$$\lim_{(x,y) \rightarrow (1,-3)} \frac{x^3 - 2y}{x^2 + y^2}.$$

i) \_\_\_\_\_

ii)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}.$$

ii) \_\_\_\_\_

5. Let  $f(x, y) = xy^2 \cos(xy)$ .

Find the indicated partial derivatives and circle your answer:

i)  $\frac{\partial f}{\partial x}$  or  $f_x$

ii)  $\frac{\partial^2 f}{\partial y^2}$  or  $f_{yy}$

6. Let  $f(x, y) = xy + \frac{x^2}{y}$ .

i) Find an equation in standard form (i.e.  $Ax + By + Cz = D$ ) of the plane tangent to the surface  $z = f(x, y)$  when  $(x, y) = (2, -1)$ .

i) \_\_\_\_\_

ii) Find the directional derivative of  $f(x, y)$  at  $(-1, 1)$  in the direction of  $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j}$ .

ii) \_\_\_\_\_

7. Let  $f(x, y) = \frac{1}{2}x^2 + y^2 + x^2y + 13$ .

Find and classify all critical points (as local maxima, local minima or saddle points) of  $f(x, y)$ . Circle your answers.

8. Use the method of Lagrange multipliers to find the minimum value of the function  $f(x, y) = x^2 + 4xy + 2y^2$  on the line  $x - y = 7$ .

8) \_\_\_\_\_

9. Consider the iterated integral

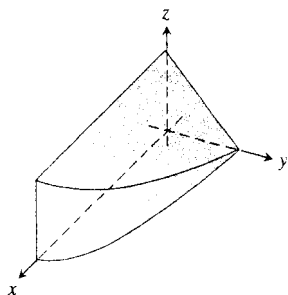
$$\int_0^3 \int_{2x}^6 \tan^{-1}(x^3) + \frac{y}{x^2} dy dx.$$

Rewrite the double integral as an iterated integral with the order of integration interchanged. **DO NOT EVALUATE EITHER INTEGRAL!**

10. Evaluate the integral

$$\iiint_S y \, dV,$$

where  $S$  is the region in the first octant bounded by the plane  $y + z = 2$  and the cylinder  $x = 4 - y^2$ .



10) \_\_\_\_\_

11. Use a triple integral to find the **mass** of the solid above the  $xy$ -plane below the surface  $z = 1 - x^2 - y^2$  with density function  $\delta(x, y, z) = z$ . It may be easier to change to a different coordinate system.

11) \_\_\_\_\_