Final Exam for MAT 296 Spring 2005

READ THIS BEFORE YOU BEGIN!

This examination contains 12 problems on 11 pages for a total value of 200 points. Point values for each problem are indicated. It is your responsibility to make sure that all problems and pages are present!

To receive full or partial credit, you MUST show correct work leading to the correct answer. Unsupported answers will receive little or no credit.

Non-symbolic graphics calculators may be used in this examination. Calculators capable of symbolic computations, such as the TI-89 or TI-92 may NOT be used.

Print Your Name	Instructor's Name

Your signature: _____

DO NOT WRITE IN THE TABLE BELOW

Problem	Score
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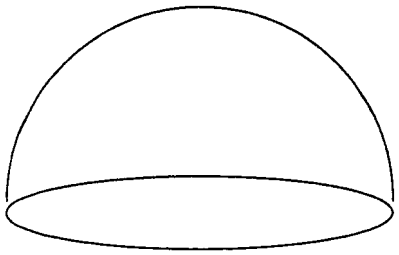
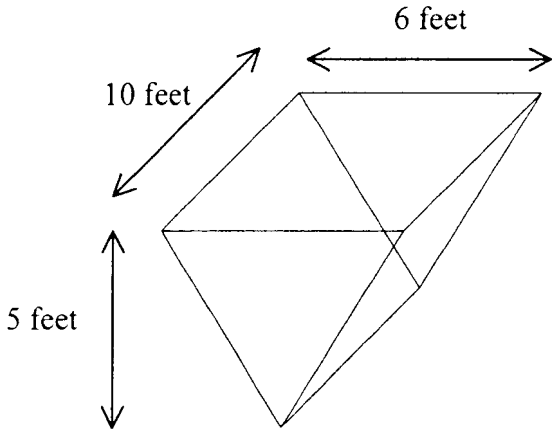
(1) [15 pts] As shown below, a hemispherical tank of radius 10 feet is filled with water to a depth of 5 feet. Set up an integral that can be used to find the volume of water in the tank. Do NOT evaluate the integral.



(2) [15 pts] Consider the region in the first quadrant bounded by $y = x^3$, x=1, and the x-axis. Set up an integral that can be used to find the volume of the solid obtained by rotating the region about the line x=2. Include a carefully labeled sketch. Do NOT evaluate the integral.

(3) [8 pts] Set up an integral that can be used to find the length of the curve given by $y = x^{3/2}$ from the point (1,1) to the point (4, 8). Do NOT evaluate the integral.

(4) [15 pts] A tank whose vertical cross section is a triangle is shown below. Assume that the tank is 10 feet long and is full of water and that the water is to be pumped to a height 10 feet above the top of the tank. Set up an integral that can be used to find the work done in emptying the tank. The weight of water is 62 pounds per cubic foot. Do NOT evaluate the integral.



(5) [16 pts] Determine each of the following limits.

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$$

$$\lim_{x \to \infty} x^{1/x}$$

(6) [24 pts] Evaluate each of the following integrals:

$$\int x^2 e^x \, dx$$

$$\int \frac{1}{x^2 + x - 2} \, dx$$

(7) [30 pts] Evaluate each of the following integrals:

$$\int \tan^{-1}(x)\, dx$$

$$\int \frac{1}{(1-x^2)^{3/2}}\, dx$$

(8) [24 pts] Determine whether each of the following integrals converges or diverges. If it converges, find the value of the integral. If it diverges, justify why.

$$\int_1^{\infty} \frac{\ln(x)}{x}\, dx$$

$$\int_0^1 \frac{1}{\sqrt{1-x}}\, dx$$

(9) [18 pts] Determine whether each of the following series converges or diverges. You MUST justify your answer!

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n}$$

$$\sum_{n=1}^{\infty} \frac{n}{n+1}$$

$$\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^2}$$

(10) [10 pts] Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is absolutely convergent, conditionally convergent or divergent. You MUST justify your answer!

(11) [15 pts] Consider the following power series:

$$\frac{(x+1)}{1\cdot 2}+\frac{(x+1)^2}{2\cdot 2^2}+\frac{(x+1)^3}{3\cdot 2^3}+\frac{(x+1)^4}{4\cdot 2^4}+\ldots$$

(a) Find a formula for the n-th term and write an expression using sigma-notation (Σ) for this power series.

(b) Find the convergence set of the power series. Be sure to check convergence at the endpoints of the interval (if appropriate). You MUST justify your answer.

(12) [10 pts] Find the first three terms of the Maclaurin series of $f(x) = \dfrac{1}{\sqrt{1-x}}$, for $|x| < 1$.