

MAT 296	Final Exam	May 3, 2004
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Name: \_\_\_\_\_

Instructor: Coman, Kim, Lutoborski, Shaw, Zacharia (Circle one)

**INSTRUCTIONS**

- There are a total of 13 problems, total value of 200 points (11 pages). It is your responsibility to make sure that all 13 are present.
- **Show all your work.** Minimal credit will be given for answers without supporting work.
- Calculators without symbolic capabilities are allowed.
- You should **simplify** your answers when appropriate.

Do not write below this line

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#	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7(a)	16	
7(b)	16	
8	12	
9	12	
10(a)	12	
10(b)	12	
10(c)	12	
11	12	
12	12	
13	12	
Total	(200)	

1. (12 pts) Find the volume of a solid, resembling a football, obtained by rotating the ellipse  $9x^2 + 2y^2 = 18$  about the  $y$ -axis.

2. (12 pts) Find the volume of the solid obtained by rotating the region bounded by the  $x$ -axis and the curve  $y = \sin(x^2)$ ,  $0 \leq x \leq \sqrt{\pi}$ , about the  $y$ -axis.

3. (12 pts) Find the length of the curve  $y = \frac{2}{3}x^{\frac{3}{2}} + 15$ ,  $0 \leq x \leq 15$ . Simplify your answer.

4. (12 pts) A 12 inch spring is stretched to 15 inches by a force of 75 pounds.
- (a) What is the spring constant in pounds per inch?
  - (b) Find the work done in stretching the spring.
  - (c) Find the extra work to stretch it 3 more inches.

5. (12 pts) Find the limit

$$\lim_{x \rightarrow 0} \frac{e^{x - \sin x} - 1}{x - \sin x}$$

6. (12 pts) Find the limit

$$\lim_{x \rightarrow \infty} x \left( \tan^{-1} x - \frac{\pi}{2} \right)$$

7. Evaluate each of the following integrals

(a) (16 pts)  $\int x \tan^{-1} x \, dx$

(b) (16 pts) (Hint: Use partial fraction decomposition)  $\int \frac{1}{x^2(1+x)} dx$

8. (12 pts) Compute the improper integral or show that it diverges

$$\int_{-4}^0 \frac{1}{\sqrt{-x}} dx$$

9. (12 pts) Compute the improper integral or show that it diverges

$$\int_e^{\infty} \frac{1}{x(\ln x)^2} dx$$

**10.** Determine whether or not the following series converge. (State your reasoning carefully, name the test(s) you use and show all steps in your work.)

(a) (12 pts)  $\sum_{n=1}^{\infty} \frac{n^5}{(n!)^2}$

(b) (12 pts)  $\sum_{n=1}^{\infty} ne^{-n^2}$

(c) (12 pts)  $\sum_{n=1}^{\infty} \frac{1}{n \sqrt[n]{n}}$

11. (12 pts) Determine the values of  $x$  for which the following series converges absolutely, converges conditionally or diverges

$$\sum_{n=1}^{\infty} \frac{3^n}{n^2} x^n$$

12. (12 pts) Find the first 4 terms of the Taylor series at 0 for the function

$$f(x) = \frac{1}{\sqrt{1-x^2}}$$

(That is find terms up through the  $x^3$  term.)

13. (12 pts) Find the power series in  $x$  for the given function  $f$ . Specify the radius of convergence of the series. (Hint: Make a suitable substitution into the geometric series)

$$f(x) = \frac{x^3}{1 - 2x^2}$$