Final Exam

Math 296 Fall 2004

Name_____

Signature_____

Circle your instructor's name: Pelley Graves Latour Griffin Meyer

READ THIS BEFORE YOU BEGIN

This examination contains 14 problems on 11 pages. Point values are indicated with a total of 200 points. It is your responsibility to make sure that all problems and pages are present. You may use a non-symbolic graphing calculator. No books or notes are allowed on this exam. Your solutions must be written legibly and contain all of the necessary steps which enabled you to arrive at your answer to receive full credit for the problem. Unsupported answers will receive little or no credit. If there is a space for your answer, write your answer in the space. Otherwise circle your final answer.

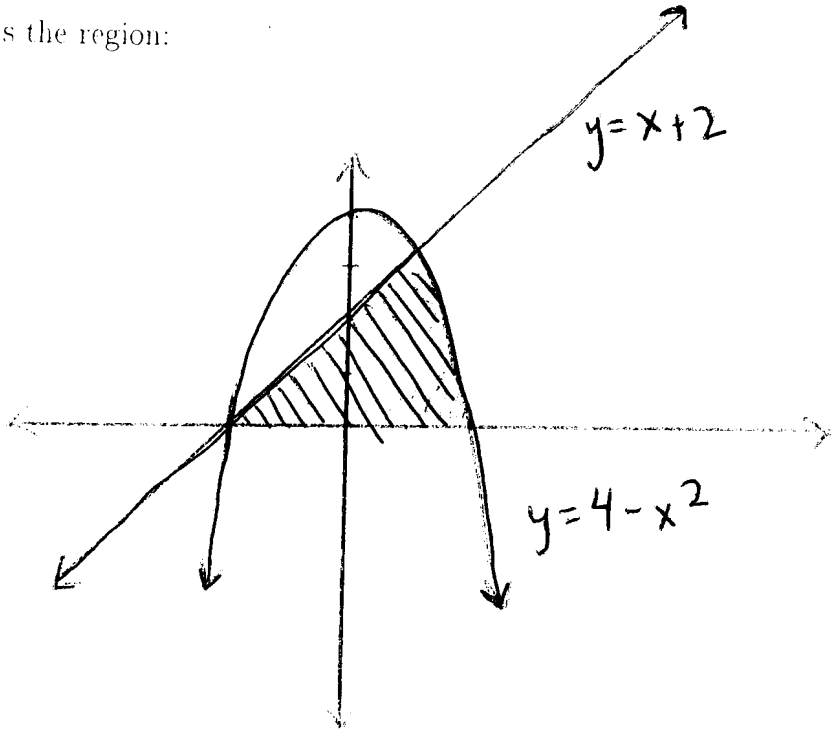
DO NOT WRITE IN THE TABLE BELOW

Problem	possible	actual
1	12	
2	12	
3	10	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	14	
11	30	
12	18	
13	12	
14	20	
Total	200	

CLARIFICATION

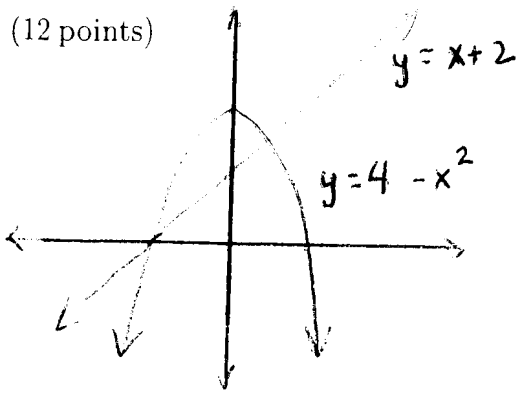
I forgot to shade the region for problem #1. Please correct my mistake on the board.

Here is the region:



1. Find the area of the shaded region.

(12 points)



1. _____

2. Let R be the first-quadrant region enclosed by the curves $y = x^4$ and $y = \sqrt{x}$. Find the volume of the solid formed by revolving R about the $y-$axis.

(12 points)

2. _____

3. Set up, but don't evaluate, the integral that gives the length of the curve $y = \sqrt{e^x + 1}$ from $x = 0$ to $x = 3$. Circle your final integral.

(10 points)

4. Integrate

$$\int \frac{1}{\sqrt{9 - 4x^2}}\, dx.$$

(12 points)

4. _____

5. Integrate

$$\int t^3 \ln t \; dt.$$

(12 points)

5. _____

6. Integrate

$$\int \frac{x^5}{\sqrt{1-x^2}}\, dx.$$

(12 points)

6. _____

7. Integrate

$$\int \frac{5x+1}{(x-1)(x+2)}\,dx.$$

(12 points)

7._____

8. Find the limit.

$$\lim_{x \to 0} \frac{x \sin x}{1 - \cos x}$$

8. _____

(12 points)

9. Use direct integration (i.e. actually do the integration) to establish if the improper integral converges or diverges. If it converges, give the value of the integral. If it diverges, briefly explain why.

$$\int_0^\infty \frac{x}{(x^2+1)^3}\, dx$$

(12 points)

10. For each of the following geometric series, determine whether they converge or diverge. Briefly explain how you know if the series converges or diverges. If the series converges, give the sum.

(7 points each part)

a)

$$\frac{1}{8} - \frac{1}{6} + \frac{2}{9} - \frac{8}{27} + \frac{32}{81} - \cdots$$

b)

$$\sum_{n=3}^{\infty} \frac{3^{n+1}2^n}{7^{n-4}}$$

11. Determine whether each series converges or diverges and give a correct proof (i.e. a correct explanation with an appropriate series test):

(10 points each)

a)

$$\sum_{n=2}^{\infty} \frac{n}{\sqrt{(n^3-1)}}$$

b)

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}\, 2^n}{n!}$$

c)

$$\sum_{n=1}^{\infty} \frac{\cos(3n)}{n^2}$$

12. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}} x^n .$$

Be sure to clearly communicate your answer.

(18 points)

13. Find the first four non-zero terms of the Maclaurin series of $f(x) = \dfrac{1}{\sqrt{x+1}}$. Write your answer in the form:

term1 + term2 + term3 + term4 + ...

(12 points)

14. Do each part.

a) Use the Maclaurin series

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \dots$$

to find the Maclaurin series for the function $\ln(1 + x^2)$.

(12 points)

b) Use the Maclaurin series

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

to estimate the value of $\cos(1)$ to within $\frac{1}{1000}$ of the actual value. Justify your approximation.

(8 points)