

Final Exam for MAT 296, Spring 2003

There are 10 problems in this exam worth a total of 200 points. To receive full or partial credit the correct work leading to the correct answer must be written down. Graphics calculators may be used in this examination. However, calculators capable of symbolic computations, such as TI-89 or TI-92 may NOT be used.

Your Name	Instructor's Name

Your Signature:

Problem	Score
1	
2	
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Total	

(1) Compute the following limits.

(a) (11 points) $\lim_{x \rightarrow 0} \frac{1 - \cos(x^2)}{x^2 + \tan(x^2)}$

(b) (11 points) $\lim_{x \rightarrow 0} (e^x + x)^{1/x}$

(2) (13 points) Find the length of the arc of the curve $f(x) = \frac{1}{3}(2 + x^2)^{3/2}$ on the interval $[0, 3]$.

(3) (11 points) The region in the first quadrant bounded by the lines $y = 0$, $x = 1$ and the curve $y = x^3$ is rotated about the line $x = 1$. Set up an integral to evaluate the volume of the resulting solid. Include a carefully labeled diagram. DO NOT EVALUATE THE INTEGRAL.

(4) (11 points each) Determine whether the following series converge or diverge. Be sure to justify your answers:

(a)
$$\sum_{n=1}^{\infty} \frac{3^n n}{(2n-1)n!}$$

(b)
$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

(c)
$$\sum_{n=1}^{\infty} \frac{(-2)^n + 1}{5^n}$$

(5) (11 points each) Evaluate the following integrals:

(a) $\int \frac{x^2}{x^2 + 4} dx$

(b) $\int \frac{x}{e^x} dx$

(c) $\int \sec^4 x dx$

- (6) (13 points) A tank having the shape of an inverted right circular cone of height 6 feet and base radius of 3 feet, is full of water. How much work is performed in pumping all the water in the tank over the top edge? The density of water is 62.4 lb/ft^3 .

- (7) (11 points each) Determine whether the following integrals are convergent or divergent. If convergent, find their value.

(a) $\int_0^4 \frac{x^2 + 7}{\sqrt{x}} dx.$

(b) $\int_1^{\infty} \frac{x^2 + 1}{x^3} dx.$

- (8) (16 points) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3 + 2}$ is absolutely convergent, conditionally convergent, or divergent. Be sure to justify your answers.

- (9) (21 points) Find the radius and the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n n (x-4)^n}{2^n}$. Does it converge absolutely, converge conditionally, or diverge at the endpoints of the interval of convergence?

(10) (16 points) Compute the first four nonzero terms in the MacLaurin expansion of the function $f(x) = x^3 + e^x \cos x$.