

MAT 296 FINAL
Spring 2002

NAME: _____

INSTRUCTOR: _____

This examination has 10 problems. It is your responsibility to make sure that all are present.

Show **ALL** work. Minimal credit will be given for answers without supporting work.

A graphics calculator may be used on this examination. However, a symbolic calculator, such as the TI-89 or TI-92, may NOT be used.

Do Not Write Below

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

TOTAL _____

(1) [15 pts] Determine each of the following:

(a) $\lim_{x \rightarrow 0} \frac{e^x - 1}{\ln(x + 1)}$

(b) the limit of the sequence $a_n = \left(1 - \frac{2}{n}\right)^{3n}$

(2) [10 pts] Consider the region in the first quadrant bounded by $y = x^2$ and $y=x$.

(a) Draw a carefully labeled diagram and set up an integral to find the volume when this region is rotated about the x-axis. Do NOT evaluate the integral.

(b) Draw a carefully labeled diagram and set up an integral to find the volume when this region is rotated about the y-axis. Do NOT evaluate the integral.

(3) [15 pts] A tank has the shape of a paraboloid obtained by revolving the curve $y = x^2$, $0 \leq x \leq 3$, about the y -axis. x and y are measured in feet. The tank is initially full of water. Find the work needed to pump the water to a height 10 feet above the tank. The water weighs 62.4 pounds per cubic foot.

(4) [20 pts] Evaluate each of the following integrals.

(a) $\int \frac{1}{(x^2 + 1)^{\frac{3}{2}}} dx$

(b) $\int \frac{2x}{x^2 - 5x + 6} dx$

(Hint: use partial fractions)

(5) [20 pts] Evaluate each of the following integrals.

(a) $\int \cos^3(x) dx$

(b) $\int x \sin(x) dx$

(6) [20 pts] Determine if the following improper integrals converge and, if they do converge, evaluate them.

(a) $\int_1^4 \frac{1}{\sqrt{x-1}} dx$

(b) $\int_{-\infty}^{\infty} x^2 e^{-x^3} dx$

(7) [10 pts] Does the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converge or diverge? Give reasons for your answer!

(8) [10 pts] Determine whether this series converges absolutely, converges conditionally, or diverges. Give reasons for your answer! $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 23}{2n+3}$

(9) [15 pts] Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{2^n}{n^2} x^n$. Does it converge absolutely, converge conditionally or diverge at the endpoints of the interval of convergence?

(10) [10 pts] Compute the first three non-zero terms in the Maclaurin expansion of $f(x) = x + e^{-x} \sin x$