

MAT 296, Fall 2002
FINAL EXAM

Name: _____ (Please print)

Instructor: Barth, Coman, Cox, Meyer, Siagiova (Circle one)

INSTRUCTIONS

- There are a total of 10 problems. It is your responsibility to make sure that all 10 are present.
- **Show all your work.** Minimal credit will be given for answers without supporting work. In particular, for infinite series problems, you must specify by name the “test(s)” you use and check that all necessary conditions are satisfied.
- A scientific graphics calculator may be used on this final. However, a symbolic calculator, such as a TI-92, may **not** be used.
- Please **simplify** your answers when appropriate.
- Please keep your answers **exact** unless an approximation is called for. (For example, π and **not** 3.1415, $\sqrt{2}$ and **not** 1.414.)

#	Points	Score
1	(10)	
2	(10)	
3	(8)	
4	(12)	
5	(12)	
6	(12)	
7	(12)	
8	(8)	
9	(8)	
10	(8)	
Total	(100)	

1. (10 points) Determine each of the following:

(a) $\lim_{x \rightarrow 0} \frac{e^{3x} - 1 - 3 \sin x}{\cos x - 1}$

(b) the limit of the sequence $a_n = (1 + 3 \sin(1/n))^n$.

2. (10 points) Consider the region in the first quadrant bounded by $x = 0$, $y = \sqrt{x}$, $y = 2$.
- (a) Sketch and label this region and set up an integral to find the volume when this region is rotated about the x -axis. Be sure to simplify the integrand, but do NOT evaluate the integral.

(b) Sketch and label this region and set up an integral to find the volume when this region is rotated about the y -axis. Be sure to simplify the integrand, but do NOT evaluate the integral.

3. (8 points) A water tank has the shape of a cone obtained by revolving the curve $y = 2x$, $0 \leq x \leq 2$, about the y -axis. The variables x and y are measured in feet. The tank is initially full of water. Find the work needed to pump the water to a height 10 feet above the top of the tank. The water weighs ρ pounds per cubic feet, so your answer should be given in terms of ρ .

4. (12 points) Evaluate each of the following integrals:

(a)
$$\int \frac{x^3}{\sqrt{4-x^2}} dx$$

(b)
$$\int \frac{5x}{x^2+3x-4} dx$$
 Hint: partial fractions.

5. (12 points) Evaluate each of the following integrals:

(a)
$$\int_0^{\pi/4} \frac{\sin \theta}{1 + \cos \theta} d\theta$$

(b)
$$\int x e^{-3x} dx$$

6. (12 points) Determine which of the following improper integrals are convergent and which are divergent. Evaluate the convergent one(s).

(a)
$$\int_0^4 \frac{dx}{(x-2)^2}$$

(b)
$$\int_0^{\infty} \frac{x}{(1+x^2)^{5/2}} dx$$

7. (8 points) Determine whether or not the following infinite series converges. Be sure to state your reasoning carefully, naming any “tests” you use and showing that you have checked to see the test applies.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln(n^2))^2}$$

8. (12 points) Determine whether the following infinite series converges absolutely, converges conditionally, or diverges. Be sure to state your reasoning carefully, naming any “tests” you use and showing that you have checked to see the test applies.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 1000}{3 + 7n}$$

9. (8 points) Find the convergence set of the following series. Be sure to state your reasoning carefully, naming any “tests” you use and showing that you have checked to see the test applies.

$$\sum_{n=1}^{\infty} \frac{x^n}{n5^{n+1}}$$

10. (8 points). Compute the Maclaurin series for the function

$$f(x) = \sqrt{1 + 2x}$$

but only through the x^2 term.