

MATHEMATICS 295  
FINAL EXAMINATION – SPRING 2004

Print Your Name \_\_\_\_\_

Signature \_\_\_\_\_

Print Your Instructor's Name \_\_\_\_\_

Section # \_\_\_\_\_, Recitation Section # \_\_\_\_\_

Student Identification Number \_\_\_\_\_

**INSTRUCTIONS.** This examination has 11 problems and 10 printed pages. (There is 1 additional page for scrap work.) **Make sure your examination copy is complete before you begin work.**

There are 200 points available on this examination. The point values are indicated for each of the 11 problems.

**All work for which you seek credit must be written on the printed pages in the appropriate places.** The last scrap page of this booklet will not be graded! **All answers must be justified. Calculators may be used to check your answers but not to justify them.** Calculators that have symbolic manipulation capabilities like the TI-89 or TI-92 are prohibited.

Do not write below this line

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6. \_\_\_\_\_

Total \_\_\_\_\_

1. (5 points each) Evaluate the following limits. If a limit does not exist, say so and explain why not:

a)  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 1}$

b)  $\lim_{x \rightarrow 0} \frac{3x \tan x}{\sin x}$

2. (5 points each) Let  $f(x) = \frac{2(x-1)(x+1)}{7x^2}$ . Determine the following four limits and then answer the two questions at the end.

a)  $\lim_{x \rightarrow +\infty} f(x)$

b)  $\lim_{x \rightarrow -\infty} f(x)$

c)  $\lim_{x \rightarrow 0^+} f(x)$

d)  $\lim_{x \rightarrow 0^-} f(x)$

e) Does  $f$  have any horizontal asymptote(s)? Why? If yes, what are they?

f) Does  $f$  have any vertical asymptote(s)? Why? If yes, what are they?

3. (10 points each) Find the derivatives of each of the following functions. **Do not simplify.**

a)  $f(x) = x^\pi + \frac{1}{x^{11}} + \frac{\cos x}{x^7 + 1}$ .

b)  $f(x) = e^x(\ln(2 + \sin x))$

c)  $f(x) = \tan(x^2) + \sec(\sqrt{x})$

d)  $f(x) = (x^2 + 1)^x$ . Hint: Use logarithmic differentiation.

4. (10 points) Find the equation of the tangent line to the curve:  $x^2y^2 + 4x = 10y - 12$  at the point  $(1, 2)$ .

5. (8 points) Use linear approximation (differentials) to approximate  $f(x) = 7x^3 + 3x^2 + 1$  at  $a = 1$ . Use it to estimate  $f(x)$  at  $x = 1.01$

6. (15 points) An airplane flying west at 300 miles per hour goes over the control tower at noon, and a second airplane at the same altitude, flying north at 400 miles per hour, goes over the tower one hour later. How fast is the distance between the airplanes changing at 2:00 PM ?

7. (15 points) A farmer wishes to construct a walled rectangular pen for livestock. He can build three walls using materials costing \$ 15 per foot. One wall, the western wall, must be sturdier and higher to deal with wind and snow and requires materials costing \$ 25 per foot. If he has \$ 4800 available for this project, what are the dimensions of the pen of greatest area that he can enclose. For full credit you must justify your answer using either the first or second derivative test.

8. (5 points each) Consider the following function:  $f(x) = 3x^4 + 4x^3 - 15$ .

a) Find  $f'(x)$  (the first derivative of  $f$ ).

b) Determine all the critical values of  $x$  (stationary points) for the function  $f(x)$  and the value of  $f(x)$  at each such  $x$

c) Find the absolute maximum and absolute minimum of  $f$  on the interval  $[-3, 1]$  and where each one occurs.

d) Determine the intervals where  $f$  is decreasing or increasing; find any local maxima and minima of  $f$ .

e) The second derivative of  $f$  is given by  $f''(x) = 36x(x + \frac{2}{3})$ . (Do NOT verify this. You will run out of time if you do.) Determine the intervals where  $f$  is concave up or down, and find any inflection points of  $f$ .

f) Using the ABOVE, sketch the graph of  $f$  on the interval  $[-3, 1]$  showing and LABELING all of its interesting features. NO credit will be given for only copying a picture from your graphing calculator.

9. (9 points) Find the derivative of

$$f(x) = \int_0^{x^2+x} \sqrt{2t + \sin t} dt.$$

10. (11 points each) Evaluate the following indefinite integrals:

a)  $\int \frac{x^4}{(x^5 + 6)^3} dx$

b)  $\int \frac{e^x}{2 + e^x} dx$

11. (11 points) Evaluate the definite integral:  $\int_0^{\frac{\pi}{6}} \sin^3 x \cos x \, dx$