

Disclaimer: The exam is a
Sample only of problems you might
see on your
final.

MAT 295 Calculus I
Final Examination
Fall 2004

Print your name _____

Signature _____

SU ID number _____

Print your instructor's name _____

Instructions. This examination has 14 problems and 12 printed pages. Make sure your examination is complete before you begin work.

This examination is worth 200 points. The point values are indicated for each of the problems.

Show your work clearly. All answers must be justified. Calculators may be used to check your answers, but not to justify them. Calculators that have symbolic manipulation capabilities (such as the TI-89 or TI-92) are prohibited.

Do NOT write below this line!

1. _____

8. _____

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11. _____

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12. _____

6. _____

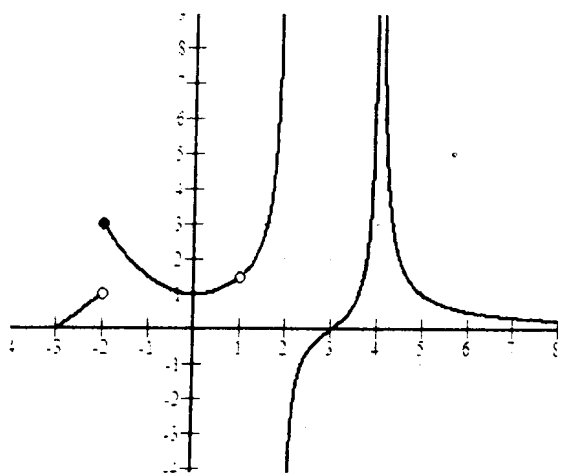
13. _____

7. _____

14. _____

Total _____

(1) [12 pts] Given the graph of $f(x)$ below, find each of the following limits, if it exists. If the limit does not exist, write DNE.



(a) $\lim_{x \rightarrow -2^-} f(x)$

(b) $\lim_{x \rightarrow 1} f(x)$

(c) $\lim_{x \rightarrow 2^+} f(x)$

(d) $\lim_{x \rightarrow \infty} f(x)$

(2) [16 pts] Evaluate each of the following. Write DNE if the limit does not exist.

(a) $\lim_{x \rightarrow 0} \frac{2x^2 - 3x}{\sin(x)}$

(b) $\lim_{x \rightarrow 10^+} \frac{|x - 10|}{x - 10}$

(c) $\lim_{x \rightarrow \infty} \frac{3x^3 + 100x}{2x^2 - 8x + 7}$

(d) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

(3) [8 pts] Given $f(x) = \begin{cases} 3x - 6, & x \leq 7 \\ -4x + a, & x > 7 \end{cases}$

For what value of a is f continuous at $x=7$? Justify your answer completely.

Include a sketch of the graph of the function.

(4) [10 pts] Using the definition of the derivative, find $f'(x)$ for $f(x) = 3x^2 - 2x$

(5) [15 pts] Find the equation of the tangent line to the curve $2x^2y^2 + 4x = 3y^2$ at the point (1,2).

(6) [15 pts] A ladder 20 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 feet per second, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 12 feet from the wall?

(7) [36 pts] Find the derivative of each of the following functions. Do NOT simplify your answer.

(a) $y = \frac{1}{\sqrt{x^2 + 1}}$

(b) $f(x) = \frac{x+1}{2x-3}$

(c) $y = \sec(5x^3)$

(d) $H(x) = \cos^3(5x)$

(e) $g(x) = x^2 e^{3x}$

(f) $F(x) = \int_1^{x^2} \sin^2(t) dt$

(8) [8 pts] Using logarithmic differentiation, find the derivative of $f(x) = \frac{x^3(x-8)^4}{(x^2+9)^6}$

(9) [8 pts] Sketch the graph of a function $f(x)$ having the following characteristics:

$$f(0) = 4 \text{ and } f(6) = 0$$

$$f'(x) < 0 \text{ if } x < 2 \text{ and } x > 4$$

$$f'(2) \text{ does not exist}$$

$$f'(4) = 0$$

$$f'(x) > 0 \text{ if } 2 < x < 4$$

$$f''(x) < 0 \text{ if } x \neq 2$$

(10) [15 pts] Consider the function $f(x) = 2x^3 - 21x^2 - 48x + 6$ on the closed interval $[-2,6]$. Be sure to justify each of your answers!

(a) Find the absolute maximum and absolute minimum values of the function on $[-2,6]$

(b) Where on $[-2,6]$ do these absolute maximum and minimum values occur?

(c) Where on $[-2,6]$ is the function increasing?

(d) Where on $[-2,6]$ is the function concave down?

(11) [15 pts] From an 8 by 15 rectangular piece of posterboard, cut four identical squares, one in each corner. What size squares should be cut out to achieve the maximum volume of the open box obtained by folding up the sides?

Include a carefully labeled diagram as part of your solution. Justify your answer using the first or second derivative test.

(12) [7 pts] Use a Riemann Sum with $n=4$ subintervals and right hand endpoints to estimate $\int_1^3 x^2 dx$.

(13) [7 pts] Find the mean (or average) value of the function $f(x) = |9 - x|$ on the closed interval $[6, 11]$.

(14) [28 pts] Evaluate each of the following:

(a) $\int_0^{\frac{\pi}{2}} \sin^3(x) \cos(x) dx$

(b) $\int \frac{x^4 - 8}{x^2} dx$

(c) $\int_0^1 x \sqrt{x^2 + 1} dx$

(d) $\int e^{4x} + \frac{3}{x} dx$