

MATHEMATICS 295
FINAL EXAMINATION - FALL 2003

Print Your Name _____

Signature _____

Print Your Instructor Name _____

Section # _____, Recitation Section # _____

INSTRUCTIONS. This examination has 13 problems and 10 printed pages. (There are 2 additional pages for scrap work.) **Make sure your examination is complete before you begin work.**

There are 200 points available on this examination. The point values are indicated for each of 13 problems.

All work for which you seek credit must be written on the printed pages in the appropriate places. The last two scrap pages of this booklet will not be graded! **All answers must be justified.**

Do not write below this line

1. _____	8. _____
2. _____	9. _____
3. _____	10. _____
4. _____	11. _____
5. _____	12. _____
6. _____	13. _____
7. _____	Total _____

1. (8 points each) Evaluate the following limits:

a) $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 + x - 6}$

b) $\lim_{x \rightarrow \infty} \frac{7x^5 + 3x^3 + 4}{2x^5 - 2x^2 - 3x}$

c) Given $\frac{\sin x}{x} \leq f(x) \leq 1 + x$, find $\lim_{x \rightarrow 0} f(x)$

2. (8 points each) Find the derivatives of the following functions:

a) $f(x) = \frac{\sin x}{x^2}$.

b) $f(x) = e^x \sin x^2$

c) $f(x) = x^{x^2}$

d) $f(x) = \int_1^{\ln x} \frac{\sin t}{t} dt$.

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3. (8 points) Find the points of discontinuity, if any, of the function

$$f(x) = \begin{cases} \frac{|2x-2|}{x-1}, & x \neq 1, \\ 0, & x = 1. \end{cases}$$

4. (8 points) State the definition of the derivative and use it to find the derivative of $f(x) = 2x^2 - 3x$.

5. (8 points) Find the equation of the tangent line to the curve $x^2y^2 + x^2 - 2y^3 = 0$ at the point $(1, 1)$.

6. (8 points) Use logarithmic differentiation to find the derivative of $y = \frac{\sqrt{x}(x^2 + 1)^{17}}{(x^3 - 1)^{10}}$.

7. (8 points) A function $f(x)$ is differentiable on the interval $[-1, 1]$ and has the value 0.32 at $x = 0.25$. The derivative of f at $x = 0.25$ is equal to 1.24. Use the linearization to estimate the value of f at $x = 0.23$.

8. (10 points) A function $f(x)$ is equal to 3 at $x = 2$ and the derivative of $f'(2) = 3/7$. Find the derivative of the inverse function $f^{-1}(x)$ at $x = 3$.

9. (10 points) Water is pumped into a conical reservoir at the rate of 8π ft³/min. The reservoir stands point down. It has a height of 12 ft and the radius of its circular top is 6 ft. How fast is the water level rising when the water is 4 ft deep? The volume of the cone is $V = \frac{1}{3}\pi r^2 h$.

10. (10 points) Write down (do not evaluate) the Riemann sum R_4 for $f(x) = \sin x$ on the interval $[0, 4]$ by dividing the interval into 4 equal subintervals and using left hand endpoints as sample points.

11. (10 points) An open rectangular box that holds 36000 cubic inches is to have a rectangular base that is twice as long as it is wide. What dimensions for the box will require the least material?

12. (8 points each) Consider the following function and the given formulas for its first and second derivatives: $f(x) = \frac{x^2}{x^2 + 3}$, $f'(x) = \frac{6x}{(x^2 + 3)^2}$, $f''(x) = \frac{18(1 - x^2)}{(x^2 + 3)^3}$.

a) Determine intervals where f is decreasing or increasing; find local maxima and minima of f ;

b) determine intervals where f is concave up or down, find inflection points of f ;

c) find any vertical and any horizontal asymptotes.

13) (10 points each) Evaluate the following indefinite and definite integrals:

a) given that $\int_{-1}^{10} f(x) dx = 10$ and $\int_{-1}^2 f(x) dx = -2$, find $\int_2^{10} f(x) dx$;

b) $\int_1^4 \frac{x^3 + 2x}{\sqrt{x}} dx$

c) $\int x \sin x^2 dx$

d) $\int \frac{x^2 + 1}{x^3 + 3x + 6} dx$