Final Exam MAT 295 Fall 2002

To receive full or partial credit the correct work leading to the correct answer must be written down.

Your Name	Instructor's Name

Your Signature:

Problem	Score
1	
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Total	

(1) Compute the limits.

(a) (4pts) $\lim_{x \to 0} \dfrac{1 - \cos(x)}{\sin(x)}$

(b) (4pts) Given $3 \leq f(x) \leq x^2 + 6x + 12$ find $\lim_{x \to -3} f(x)$

(2) (6pts) Determine the constant c so that $f(x)$ is continuous at $x = 1$ where $f(x) = \begin{cases} \ln(x) & 0 < x \leq 1 \\ e^{x-1} + c & x > 1 \end{cases}$

(3) Differentiate **do not simplify**

(a) (4pts) Find $\dfrac{d}{ds}\,\dfrac{3s^{-1}+6}{2s^{1/2}+3}$.

(b) (4pts) $r(\theta)=\sin(\cos(\theta^2))$. Find $\dfrac{d}{d\theta}r(\theta)$.

(c) (4pts) Use the properties of logarithms first **and then differentiate** to find $\dfrac{d}{dx}\ln\left(\dfrac{\sin^4(x)}{x^2(x+5)^{10}}\right)$

(d) (4pts) Find $\dfrac{d}{dx}\displaystyle\int_0^{2x}\frac{t^2-4}{1+\cos^2(t)}\,dt$

(e) (4pts) Find $\dfrac{d}{dx}(\sin(x))^x$

(4) (5pts) Find the slope of the tangent line to the curve $x^2y^2 + x^3 + y^3 = 3$ at the point $(1, 1)$

(5) (5pts)The circumference of a sphere is measured to be 86 cm with a maximum possible error of 0.5cm. Estimate the relative error in calculating the volume of the sphere ($V = \frac{4}{3}\pi r^3$) with this measurement.

(6) (5pts)The top half of an hour glass is an inverted cone with height 4 cm and **diameter** 2 cm. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Sand is running out of this cone at the rate of .01 cm^3/sec. How fast is the depth of sand in this cone decreasing when the depth is 3 cm?

(7) (6pts)A rectangle is inscribed with its base on the x-axis and its upper corners on the top half of the circle $y = \sqrt{4 - x^2}$. What is the largest possible area for such a rectangle? Be sure to show that your answer is a maximum.

(8) You are given the function $f(x) = \dfrac{(x-1)^3}{x^3}$ along with its first derivative $f'(x) = \dfrac{3(x-1)^2}{x^4}$ and second derivative $f''(x) = -6\,\dfrac{(x-1)(x-2)}{x^5}$.

(a) (2pts) Find all vertical asymptotes. You must take the appropriate limits to receive credit.

(b) (2pts) Find all horizontal asymptotes. You must take the appropriate limits to receive credit.

(c) (2pts) Find all critical points. Show your work.

(d) (3pts) List all intervals on which f is increasing. Show your work.

(e) (3pts) List all intervals on which f is concave down. Show your work.

(f) (2pts) Find all inflection points. Show your work.

(9) (6pts) Calculate the Riemann sum R_n for $f(x) = 3x^2 + 1$ on the interval $[0, 3]$ by dividing the interval into n equal width subintervals and using right hand endpoints as sample points.

(10) (5pts) On the interval $[0, \pi]$ the function $f(x) = \cos(x)$ is one-to-one, so it has an inverse function $f^{-1}(x)$ whose domain is $[-1, 1]$. Find $(f^{-1})'(0)$, i.e. the derivative of the inverse function at 0.

(11) Compute the definite and indefinite integrals.

(a) (4pts) Given that $\int_1^{10} f(x)dx = 8$, $\int_1^4 f(x)dx = -2$, and $\int_8^{10} f(x)dx = 5$ find $\int_4^8 f(x)dx$.

(b) (4pts) $\displaystyle\int_1^4 \frac{2x^2+7}{\sqrt{x}}dx$.

(c) (4pts) $\displaystyle\int_0^{\pi/3} \cos(3\theta)\, d\theta$.

(d) (4pts) $\displaystyle\int \frac{x}{\sqrt{4x^2-8}}\, dx$.

(e) (4pts) $\displaystyle\int e^{4x} + \frac{3}{x}\, dx$.