MAT 295 UC
FINAL EXAM – FALL 2001
December 17, 2001

Name: ______________________________ SS# ______________________________

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You have exactly 120 minutes to complete this exam.

INSTRUCTIONS

a) Check to see that you have 10 pages and 11 questions. No credit will be given for questions from any missing pages.
b) Do not take the exam apart.
c) Put your name on every page.
d) Show all work. No credit will be given for answers without supporting work.
e) The number in parentheses to the left of each question is its point value.

DO NOT WRITE BELOW THIS LINE

1. ________ 6. ________ 11. ________

2. ________ 7. ________

3. ________ 8. ________

4. ________ 9. ________

5. ________ 10. ________

1. (20 points) Evaluate the following limits. If the limit does not exist describe how it does not exist.

a.) $\lim_{x \to +\infty} \frac{5x^3 + 2x^2 - x}{5x^2 + 3x + 1}$

b.) $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 5x + 6}$

c.) $\lim_{x \to 3+} \frac{x^2 + 3}{x - 3}$

d.) $\lim_{x \to 2} \frac{\sqrt{3x - 5} - 1}{x - 2}$

2. (15 points)

a.) State the definition of the derivative, *f ' (x)*, for a function *f(x)*.

b.) Use the definition to compute the derivative of $f(x) = \frac{3}{x}$

c.) Find an equation for the tangent to the curve $f(x) = \frac{3}{x}$ at x = 3.

3. (20 points) Find the derivatives of the following functions:

a.) $f(x) = x\cos x + \dfrac{x+1}{x+2}$

b.) $f(x) = \ln\left(\dfrac{e^x}{x+1}\right)$

c.) $y = 2x\sin\sqrt{x}$

d.) $y = x^{3/2} + x^{-8}$

4. (10 points) Find ${dy}/{dx}$ of the equation $(3xy+7)^2 = 6y$

5. (10 points) Solve the inequality $|2x+5| \le 2$

6. (12 points) The volume of a spherical balloon of radius r is $V = \frac{4}{3}\pi r^3$ and the surface area is $S = 4\pi r^2$

a.) If the balloon is inflacted at the rate of 36 ft^3/min, how fast is the **radius** increasing when r = 5 feet?

b.) How fast is S increasing when r = 5 feet?

7. (28 points) Consider the function and its derivatives:

$$f(x) = -\frac{1}{12}x^4 + 2x^2 + 3$$

$$f'(x) = -\frac{1}{3}x^3 + 4x$$

$$f''(x) = -x^2 + 4$$

There will be NO CREDIT of **calculus** work is now shown.

a.) Determine the intervals where f is increasing or decreasing.

b.) Find all x-values at which local maxima and minima occur.

c.) Determine intervals where f is concave up or down.

d.) Find the x-values where the **absolute** maximum and minimum for f occur when f is restricted to the interval [-12, 12].

8. (20 points) Compute:

a.) $\int\left(3x^2 + x^{-\frac{1}{2}} - 2\right)dx$

b.) $\frac{d}{dx}\int_0^{x^2} \ln\left(t^2 + 1\right)dt$

c.) $\int \sin x \cos^2 x \, dx$

d.) $\int_0^4 \frac{x dx}{x^2 + 9}$

9. (15 points) A rectangular plot of land will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 meters of wire at your disposal, what is the largest area you can enclose?

10. (20 points) Find the **total** area between the curve

$$y = x^3 - 9x, \quad -3 \le x \le 3$$

and the x-axis.

11. (20 points) The cups of a water cooler are conical in shape with a 3-inch radius and 6 inches in height. After being filled, the bottom of the cup springs a leak and water escapes at a rate of 1.8 cubic inches per second. How fast is the water level in the cup falling when the depth is 4 inches?