MAT286 Final Exam
May 9, 2005

1. Integrate the following indefinite integrals:

   (a). $\displaystyle\int \left(\frac{1}{x^2} + \frac{3}{x} - x^4 + \sin(x) - \tan(x)\right) dx.$

   (b). $\displaystyle\int x^3 \left(x^4 + 10\right)^{\frac{1}{4}} dx$

2. Evaluate the following definite integrals:

   (a). $\displaystyle\int_0^2 \frac{x}{x^2+1} dx$

   (b). $\displaystyle\int_0^{2\pi} \cos\left(\frac{x}{4}\right) dx$

3. An object is moving along a line with acceleration $a(t) = -2t^2 + t$ meters/hour^2. Initially, its velocity is 1 meter/hour and its position on the line is at the 3 meter mark.
(a). Find the velocity $v(t)$ of the object at time t.

(b). Find the position $s(t)$ of the object at time t.

4. (a). Set up integral(s) for the area between the curves $y = x^2$ and $y = 8 - x^2$. Make a sketch. Do **NOT** evaluate.

(b). Set up integral(s) for the area between the curves $y = x$ and $y = x^3$. Make a sketch. Do **NOT** evaluate.

5. Evaluate the following indefinite integrals:

   (a). $\int (x+1)\sin(2x)dx$

   (b). $\int 2x^2 e^{2x} dx$

6. A solid of revolution is formed by rotating about the x-axis the region bounded by the curve from $y = \sqrt{x}(4 - x^2)$ from $x = 1$ to $x = 2$. Find the volume of the solid.

7. The total cholesterol level of a patient on a special diet and medication is approximately $C(t) = 190 + 90e^{-1.6t}$, where t is in months. Find the average total cholesterol over the first 4 months of being on the special diet and medication.

8. Determine whether or not the following improper integrals converge. If the integral converges, give its value.

(a). $\displaystyle\int_1^{\infty} \frac{2x}{(x^2+1)^2}dx$

(b). $\displaystyle\int_{-\infty}^{\infty} x^2 e^{-x^3}\,dx$

9. Evaluate the following double integrals:

(a). $\iint_R \left(xy + y^2\right) dxdy$, where R is the region $1 \leq x \leq 2$, $0 \leq y \leq 1$. Make a sketch of R.

(b). $\iint_R (x + y)\, dxdy$, where R is the region $0 \leq x \leq 1$, $0 \leq y \leq x^2$. Make a sketch of R.

10. Solve the following differential equations for a general solution:

(a). $y\dfrac{dy}{dx} = x^2(2 + y^2)$

(b). $\dfrac{dy}{dt} = -ty + te^{-t^2}$

11. Set up the integral for the volume of the solid under the surface $z = x + 3y^2$ and above the rectangle $0 \le x \le 10$, $2 \le y \le 5$.

12. Initially, a tank contains 300 gallons of brine with 25 pounds of salt dissolved in it. Brine enters the tank at a rate of 4 gallons per hour and contains 2 pounds of salt per gallon. Brine leaves the tank at the rate of 4 gallons per hour.

(a). Set up the differential equation for the amount y of salt in the tank at time t.

(b). Solve the differential equation in part (a).

(c). How much salt is in the tank when $t = 150$ hours? Give units.

(d). In the distant future, how much salt is in the tank? Give units.