Final Exam
Math 285 Spring 2004

Name_____

Circle your instructor's name:

Hsiang Musto Graves

Show your work. You may use a calculator, but you must write the steps you take with the calculator. A correct answer with no work will not receive credit.

1. The growth of a bacterial colony is modeled by the function

$$A(t) = 4120e^{0.21t},$$

where A is the population t days after the initial measurement.

a) Find the population 25 days after the initial measurement. **Round your answer to the nearest whole number.**

(5,6)

1a. _____

b) When does the population first reach 18,000? **Round your answer to two decimal places.**

1b. _____

(5)

2. Look at the graph of the function below. Find the indicated limits and function values. Write "doesn't exist" if the value or limit does not exist.

(10)

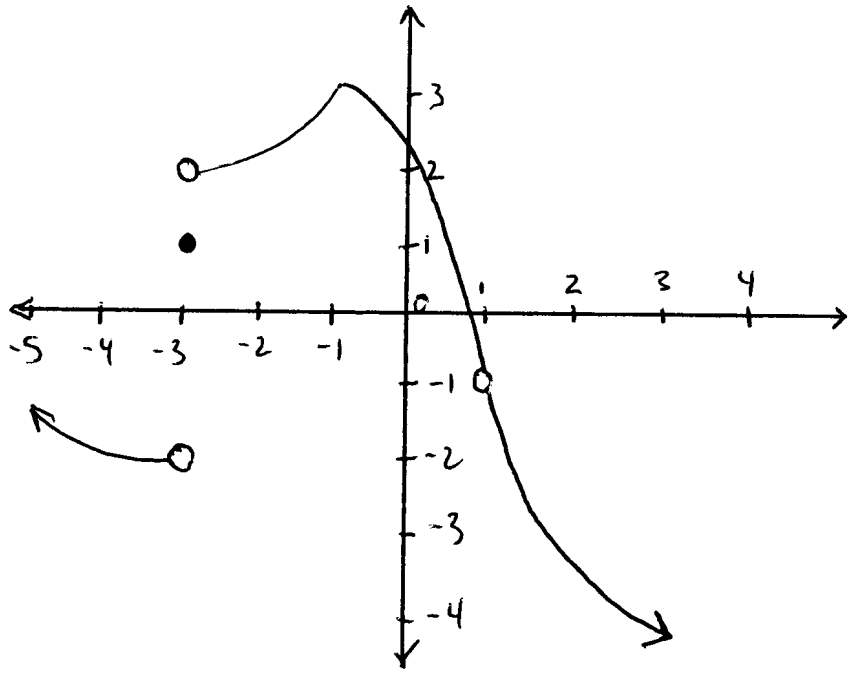
a) $\lim_{x\to 1} f(x) =$ _____

b) $f(1) =$ _____

c) $f(-3) =$ _____

d) $\lim_{x\to -3^-} f(x) =$ _____

e) $\lim_{x\to -3} f(x) =$ _____



3. Use the definition of the derivative,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h},$$

to find $f'(x)$ for

$$f(x) = x^2 + 5x.$$

(10)

3. _____

4. The hours of daylight on day number n of the year in Boston is modeled by the function

$$d(n) = 3.1\sin(0.017n - 1.377) + 12.2.$$

For each of the following values of n, use your calculator to find the derivative at that point. Make sure that your calculator is set in RADIAN mode. Circle your answer. **Round your answer to 3 decimal places.**

a) $n = 45$ b) $n = 203$

(5)

5. Find the derivative of each function. Use appropriate notation. Circle your answer. You do not need to simplify.

$f(x) = x^2\sin(3x)$ $y = x + \dfrac{1}{\sqrt{x}}$

(25)
(5 each)

5. (Continued)

$g(t) = -e^{-2t^2+1}$

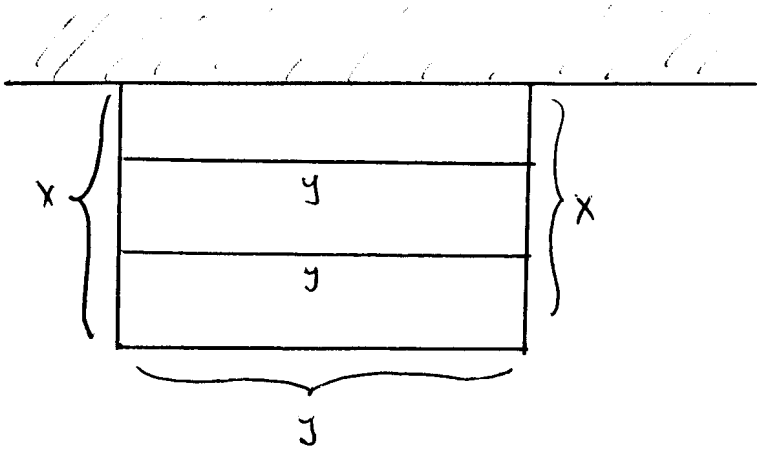
$f(x) = \dfrac{x^3+2}{\cos x}$

$g(x) = 3\ln\left(x^2 - x\right)$

6. Let $f(x) = x^3 - 6x^2 - 15x - 5$. Identify the intervals of on which the function is increasing and on which the function is decreasing, identify the intervals on which the function is concave up and on which the function is concave down and identify any relative maximums or minimums.

(10)

7. A worker needs to build a rectangular pen with dividers against a long, straight wall as shown in the diagram. The worker has 288 feet of fence. What are the dimensions of the outside of the pen that has the maximum area?



7. _____

(10)

8. a) Use implicit differentiation to find $\frac{dy}{dx}$ for

$$xy^2 + 3x^2 - 2y = 12.$$

8a. _____

(6)

b) Find the equation of the line tangent to the curve in part a) at the point $(2, 1)$.

8b. _____

(4)

9. Find $f_x(x, y)$ and $f_{yy}(x, y)$ for

$$f(x, y) = e^{-xy} - 4x^2y + 3y.$$

$f_x(x, y) =$ _____

(10)

$f_{yy}(x, y) =$ _____

10. Find and classify all critical points of

$$f(x,y) = 2x^3 + y^2 - 9x^2 + 6y - 24x + 5.$$

Circle your answers.

Hint:

The 2nd derivative test: Let $D = (f_{xx})(f_{yy}) - (f_{xy})^2$.
If $D > 0$ and $f_{xx} > 0$, then the point is a minimum.
If $D > 0$ and $f_{xx} < 0$, then the point is a maximum.
If $D < 0$, then the point is a saddle point.
If $D = 0$, then the test is inconclusive.

(10)