

Final Examination **MAT 284** Barth and Lewis
 Monday, May 9, 2005. 10:15 am – 12:15 pm

Version A

Print name:

Sign name:

Student Identification Number:

Instructor (Barth or Lewis):

TO RECEIVE CREDIT, work and reasoning should be shown for every problem except those indicated by (no work needed).

There are 32 questions (each worth 4 points) on 10 pages.

CHECK THAT YOU HAVE THE COMPLETE TEST!

Do not write below this line

Questions 1 to 9	_____	Examination # 1 (Questions 1 to 9 and 10 and 22)
Question 10	_____	
Questions 11 to 21	_____	Examination # 2 (Questions 11 to 21 and 22)
Question 22	_____	
Questions 23 to 32	_____	Examination # 3 (Questions 23 to 32 and 10)
Total	_____	

1. A company sells a product at \$11 per unit. Its fixed cost is \$15,000 and its variable cost per unit is \$7.

a) Find a function giving the company's profit $P = P(q)$ if it produces and sells q units.

$$P(q) =$$

b) At what level of production will there be a loss of \$3,000?

(Show your work)

2. If $f(s) = \frac{1}{s^3 - 2}$ and $g(s) = \sqrt{s + 1}$, then $(f \circ g)(s) =$

(You need not simplify your answers.)

3. The domain of $g(z) = \frac{1}{\sqrt{4 - 2z}}$ is

(Show your reasoning)

4. If $10^{\log(x-3)} = 7$, then $x =$

(Show your work)

5. The supply and demand equations (in some order), with prices in dollars, are:
 $p = -2q + 48$, $p = 6q + 8$. If a tax of 40¢ per unit is imposed on the supplier,

a) write the supply equation

b) write the demand equation

(No work needed)

6. The cost to produce 20 units is \$110 and the cost to produce 10 units is \$70. The cost $c = c(q)$ in dollars is a linear function of output q , i.e., c and q are linearly related. Find $c = c(q)$.

(Show your work)

7. Express $2 \log_5 x - 3 \log_5(x+1)$ as a single logarithm:

(Show your work)

8. A company has 1,000 units in stock, now selling at \$3 per unit. Next month the price will increase by \$1. The company wants total revenues from the sale of the units to be no less than \$3,600. Let x be the maximum number of units that can be sold this month. Then x must satisfy the following equation or inequality (you need not simplify or solve).
9. If the demand function is $p = 200 - 2q$ and the total cost $c = 300 + 60q$, where q is the number of units, then the profit function is.
(Show your reasoning)
10. A company holds a workshop for at least 30 people. If 30 people attend, the charge is \$200 each, and the company will reduce the charge for everyone by \$4 for each person above 30 who attends. Let x be the number of people who attend. Write the company's total revenue in dollars as a function of x . (Here $30 \leq x < 80$. Do not simplify your answer or maximize R .
(Show your work)

$$R =$$

[If you wish to let x be the number of decreases, you may do so, but check here _____ and write R below].

$$R =$$

11. $\frac{d}{dx}(5x^3 + 6) =$
(No work needed)

12. Differentiate $y = x^2 e^{(2x+1)}$ and do not simplify your answer.
(Show your work)

13. Differentiate $y = \frac{4t+2}{t^3}$ and do not simplify your answer. (Show your work)

14. Differentiate $y = \frac{5}{\sqrt{2x^3+1}}$ and do not simplify your answer.
(Show your work)

15. Find $\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x^2 - 5x + 4}$

(Show your reasoning)

16. Find the slope of the curve $y = -8x + x^4$ at $x = 1$
(Show your work)

17. Let q be the number of units, let p be the price per unit in dollars, and let the demand function be $p = 100 - 2q$. Find the marginal revenue function.
(Show your work)

18. Let $y = w^3$ and let $w = 5 - x^2$. Use the chain rule to find $\frac{dy}{dx}$ at $x = 1$ (first state the chain rule):
(Show your work)

19. Evaluate $\lim_{x \rightarrow \infty} \frac{4 + x - x^2}{2x^3 - x + 1}$.

(No work needed)

20. Differentiate $y = \ln\left[x^5\sqrt{1+x^2}\right]$

(You do not need to simplify your answer but show your work)

21. Use the DEFINITION of derivative to find the derivative of $f(x) = x^2 - 1$.
First state the definition.
(Show your work)

22. $\frac{d}{dx} [e^{\ln(4x^2+6)}] =$
(Show your work.)

23. Let $y = x^3 + 9x^2 + 24x - 7$. Find all values of x for which the curve is concave up. (Show your work and, if there are none, state "none")
24. Let $f(x) = -x^3 + 3x^2 + 24x - 8$. Find the critical points of $f(x)$. (Show your work and, if there are none, state "none".)
25. The curve $y = -x^3 + 3x^2 + 9x + 8$ has a critical point at $x = -1$. Determine if this point is a relative maximum, relative minimum, or neither. (Show your work)

26. Let $y = -x^3 + 15x^2 - 48x + 2$. Determine whether or not the point $x = 5$ is an inflection point. Justify your answer.
27. $\int \frac{2}{x^4} dx =$
(Show your work)
28. The demand function is $p = -4q + 400$, where q is the number of units and p is the price per unit in dollars. Find the output at which total revenue is the maximum.
(Prove your conclusion)
29. If $p = 800 - 2q^2$, where q is the number of units and p is the price per unit, then find the (point) elasticity of demand at $q = 10$.
(Show your work)

30. Suppose that the marginal revenue function is $500 - 12q^2$, where q is the number of units. Find the demand function.
(Show your work)

31. Presently, at \$50 per sweater, a company is selling 400 sweaters, while at \$40 per sweater it estimates that it will sell 500 sweaters. Find the (approximate) elasticity of demand.
(Show your work)

32. Let the total cost function be $c = q^2 + 3q + 400$, where q is the number of units. At what level of output q will the average cost per unit be at the minimum. [You must show that you are at the minimum.]

There are 32 questions. Be sure that you have done each one of them.