

# Final Exam

version 1

Signature \_\_\_\_\_ SU ID # \_\_\_\_\_

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Show your work. No work-no credit!

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Problem number	Possible points	Score	Subtotal
1	20		
2	20		
3	20		
4	20		
5	20		
6	20		
7	20		
8	20		
9	20		
Total	180		



1. (a) How many ways can a president, a secretary, and a treasurer be chosen from a class of 90 students? [10 points]

(b) How many ways can a class of 90 students be assigned to teams A, B, and C, with 20 assigned to team A, 30 to team B, and 40 to team C? [Give your answer in factorials.] [10 points]

2. In a carnival game, the player pays \$2.00 to play the game. The game consists of selecting an envelop from a box by the player and the player keeps the money in it. In the box, there are two envelops with \$10, three envelops with \$5, and twenty-five envelops with nothing in it. Assuming that each envelop is sealed and looks the same.

(a) Identify the random variable, and find the probability distribution. [10 points]

(b) Find  $E(X)$ . [10 points]

3. A box contains 6 good fuses and 2 defective ones. Fuses are selected one at a time (without replacement) until a good fuse is found.

(a) Draw a tree diagram that corresponding to the sequence of activities. [10 points]

(b) What is the probability that 2 fuses are selected? [10 points]

4. The heights of 11 middle school students are recorded to the nearest inch as follows

53, 60, 56, 61, 60, 64, 58, 62, 59, 57, 59.

Find the mean and the standard deviation of the heights of the 11 students. [20 points]

$\mu =$  \_\_\_\_\_ ,       $\sigma =$  \_\_\_\_\_

5. Amy deposits \$160.00 each month into a savings account that pays an annual interest rate 6% compounded monthly, the first deposit was made at the end of January, 2000 . At the end of June, 2020, what is the amount in her account? **Using Difference Equations Method to solve this problem.**

(a) What is the value of  $y_0$ ? [4 points]

$y_0 =$  \_\_\_\_\_

(b) What are the values of  $y_1, y_2, y_3$ ? [4 points]

$y_1 =$  \_\_\_\_\_

$y_2 =$  \_\_\_\_\_

$y_3 =$  \_\_\_\_\_

(c) What are the values of  $a$  and  $b$ ? [4 points]

$$a = \underline{\hspace{4cm}}$$

$$b = \underline{\hspace{4cm}}$$

- (d) Express  $y_n$  in terms of  $y_0$ ,  $a$ ,  $b$ , and  $n$ , that is, write the formula of  $y_n$ . [4 points]

$$y_n = \underline{\hspace{4cm}}$$

- (e) Using the formula from (d) to calculate the amount in Amy's account at the end of June, 2020. [4 points]

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6. Mr. and Mrs. Yogi Bear took out a bank loan of \$40,000.00 at an annual interest rate 9% compounded monthly in order to buy a house and is to be paid off the loan in 20 years.

- (a) What is their monthly payment? [10 points]

- (b) What is the total interest on the loan? [10 points]

7. (a) Transform the system of linear equations into matrix form. [5 points]

$$\begin{cases} 2x - y - 3z = -2 \\ 3x - 5y + 4z = 1 \\ 5x \quad - 3z = 3 \end{cases}$$

(b) Pivot the matrix about the circled element. [5 points]

$$\left[ \begin{array}{ccc|c} -1 & 3 & -6 & 1 \\ 1 & \textcircled{2} & -3 & -4 \\ 2 & -1 & 5 & -6 \end{array} \right]$$

(c) Perform the indicated matrix calculations.

i. 
$$\begin{bmatrix} -3 & -4 \\ -1 & -3 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 7 & -8 \\ 1 & 3 \end{bmatrix}$$

[5 points]

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ii. 
$$\begin{bmatrix} 0 & -3 & 2 \\ -1 & 8 & -5 \end{bmatrix} \begin{bmatrix} 2 & 0 & -3 \\ 0 & 7 & -2 \\ -1 & -3 & 0 \end{bmatrix}$$

[5 points]

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$$8. \begin{cases} 4x + 2y = 1 \\ 5x + 2y = -3 \end{cases}$$

- (a) Transform the above system of equations into a matrix equation of the form  $AX = B$ .  
[4 points]

$$A =$$

$$X =$$

$$B =$$

$$AX = B :$$

- (b) Find  $\Delta = \det A$ . [4 points]

$$\Delta =$$

- (c) Find  $A^{-1}$ . [4 points]

$$A^{-1} =$$

(d) Use  $A^{-1}$  which you found in (c) to solve for  $X$ . [4 points]

$$X =$$

(e) What is the solution for the system of equations (the original system of equations which is given at the beginning of this problem). [4 points]

$$\begin{cases} x = \\ y = \end{cases}$$

9. Suppose that an economy is composed of only three industries - coal, steel and electricity. Each of these industries depends on the others for some of its raw materials. Suppose that to produce \$1 of coal, it takes no coal, but \$0.03 of steel and \$0.04 electricity; to produce \$1 of steel, it takes \$0.14 of coal, \$0.05 of steel and \$0.07 of electricity; and to produce \$1 of electricity, it takes \$0.2 of coal, \$0.3 of steel, and \$0.04 of electricity. How much should each industry produce in order to be exported for consumption (not used for production) of \$2 billion coal, \$2.5 billion steel, \$3 billion electricity?

(a) Define your variables. [2 points]

$$\begin{cases} x = \\ y = \\ z = \end{cases}$$

(b) Set up the system of equations. [2 points]

(c) Transform the above system of equations, the system of equations you set up in (b), into a matrix equation of the form  $(I - A)X = D$ . [4 points]

$$A =$$

$$X =$$

$$D =$$

$$(I - A)X = D :$$

(d)  $A$  is called \_\_\_\_\_ matrix and  $D$  is call \_\_\_\_\_ matrix. [2 points]

(e) Use your calculator to find  $(I - A)^{-1}$ . [4 points]

$$(I - A)^{-1} =$$

(f) Use  $(I - A)^{-1}$  which you found in (e) to solve for  $X$ . [You may use your calculator.]  
[4 points]

$$X =$$

(g) Answer the problem in a complete sentence. [2 points]

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