

# Mat 183 Spring 2003 Final Exam

Name: \_\_\_\_\_  
( please print )

**Instructions:** Show all the work you want graded on this exam. Unsupported answers may receive little or no credit. List the values put into your calculator and the specific program being used.

## Part I – Probability

1. Area codes in the U.S. use three digit numbers where the first digit is not a 0 or a 1.

( a ) How many area codes are possible if the middle digit must be either 2 or 3 ?

Problem	Points	Score	
1	4		
2	6		
3	6		/16
4	5		
5	6		
6	7		/18
7	5		
8	5		
9	6		/16
<b>Total</b>	50		

( b ) How many area codes are possible if the middle digit is unrestricted ?

**2.** A jar contains 6 blue marbles and 7 red marbles. A random sample of five marbles is chosen without replacement.

**( a )** What is the probability that all the marbles are blue ?

**( b )** What is the probability of getting exactly 3 blue marbles and 2 red ones ?

**( c )** What is the probability that at least 3 of the marbles are blue ?

**3.**

**(a)** Complete the following table so that it is a probability distribution for the random variable  $X$ .

$k$	$\Pr(X = k)$
0	.13
1	.25
2	
3	.37
4	.09

**(b)** Find the expected value of  $X$ .

**(c)** Let  $E = \{0, 2, 3\}$  and  $F = \{1, 3\}$ . Find the conditional probability  $\Pr(E | F)$ .

## Part II – Statistics and Math of Finance

4. The weight of bowling balls produced by a factory is normally distributed with a mean of 15.1 pounds and a standard deviation of .7 pounds. Let  $X$  be a random variable whose possible values are the weights of the balls produced by the factory .

( a ) Find  $\Pr ( 14.2 \leq X \leq 16.3 )$  – Round your answer to the nearest thousandth.

( b ) If 2000 bowling balls are selected at random, approximately how many will have a weight between 14.2 pounds and 16.3 pounds ?

- 5.** You buy a car with a down payment of \$ 2000 . Your monthly payment is \$400 for 5 years, and the interest rate is 6% compounded monthly. (Use TVM-solver)

( a ) What was the price of the car ?

( b ) How much do you still owe on the car after 3 years ?

6. An initial deposit of \$12000 is made into an account paying 6% interest compounded monthly. At the end of each month, a withdrawal of \$150 is made. Let  $y_n$  be the balance in the account after  $n$  months.

( a ) Find a difference equation describing this account.

( b ) What is the balance in the account after 12 months ?

( c ) What is the *minimum* initial deposit needed so that this account becomes a **perpetuity**? (i.e. you can withdraw \$150 per month forever.) Justify your answer.

### Part III – Linear Algebra

7. Using the `rref` command find all solutions to the following system of equations.

$$\begin{cases} x + 2y - z = 3 \\ x - y + z = 4 \\ 2x + 4y - 2z = 6 \end{cases}$$

8. Let  $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -2 \\ 0 & 4 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ ,  $C = \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$ . Which of the following matrix

operations can be done? Briefly explain why or why not. You do NOT need to do the operation.

(a)  $B + C$

(b)  $AB$

(c)  $BA$

9. The economy of Arrakis consists of two industries, water and spice. The water industry consumes \$ .06 of water and \$ .10 of spice to produce \$1 of water. The spice industry consumes \$ .20 of water and \$ .15 of spice to produce \$1 of spice.

( a ) What is the Input-Output matrix for this economy ?

( b ) At what level should each industry produce in order to have \$ 18 million of water and \$ 23 million of spice available for export ?