

Name _____

**MAT 117 Final Exam
December 15, 2003**

Answer all questions.

- I A) Recall the game “What’s My Number?” The rules were that a number is picked with three distinct digits. Your job is to guess what the correct number was. Here is a list on the guesses and the information obtained for each guess. Give the number and justify why the number you have chosen is the right number.

<u>Guess</u>	<u>Correct Digits</u>	<u>Correct Places</u>
123	1	0
234	1	1
345	2	0
456	1	0
567	1	1
678	0	0
789	0	0

- B) Abigail was to plant fig trees in a rectangular array. She has 36 trees. Find all possible numbers of rows if each row (in a given arrangement) is to have the same number of trees.

II A) Mackenzie was asked to write $2 \times 12^6 + 10 \times 12^3 + 1 \times 12^0$ as a base twelve numeral. She wrote 2T1. Was she correct? If yes, why? If not, why not? Explain your reasoning and her possible reasoning.

B) Consider the numeration system in use with some machines that uses the combination of our digits and the first six letters of our alphabet. It is assumed that the letters represent ten, eleven, and so on in order. The digits in this system are thus $\{0, 1, 2, 3, \dots, 9, a, b, \dots, f\}$.

i) What is the base of this numeration system? Why?

ii) What number (in base ten) does the numeral aa represent? Why?

III A) Jana was asked to subtract in base two: $11000000_{two} - 1000001_{two}$. She showed this work:

$$\begin{array}{r} 11000002 \\ - 1000011 \\ \hline \end{array} \quad \begin{array}{r} 11000022 \\ - 1000111 \\ \hline \end{array} \quad \begin{array}{r} 11000222 \\ - 1001111 \\ \hline \end{array} \quad \begin{array}{r} 11002222 \\ - 1011111 \\ \hline \end{array} \quad \begin{array}{r} 11022222 \\ - 1111111 \\ \hline \end{array}$$

$$\begin{array}{r} 11222222 \\ - 2111111 \\ \hline \end{array} \quad \begin{array}{r} 13222222 \\ - 12111111 \\ \hline \end{array}$$

Will she be successful in subtracting? Explain her possible reasoning.

B) The table on the right indicates the operation \oplus on $\{a, b, c\}$.

\oplus	a	b	c
a	a	a	a
b	a	b	c
c	a	c	b

- i) Is the set $\{a, b, c\}$ closed under \oplus ? Explain.
- ii) Does the operation \oplus have an identity element in $\{a, b, c\}$? Explain.
- iii) Is \oplus commutative on $\{a, b, c\}$? Explain.
- iv) Does c have an inverse? Explain.

IV A) Let p , q , and r be prime numbers. Find the least common multiple of the two integers p^5qr^3 and p^2q^4 .

B) Consider the integer with prime factorization $p \times q \times r$, where p , q , and r are prime numbers. Does this integer have more than six factors, exactly six factors, or less than six factors? Justify your answer.

V True or False? If true, explain why. If false, explain why OR give a counterexample.

A) A divisibility test for five in base five need only consider the last digit in the numeral.

B) There is a number that is both prime and composite.

C) All real numbers are natural numbers, whole numbers or integers.

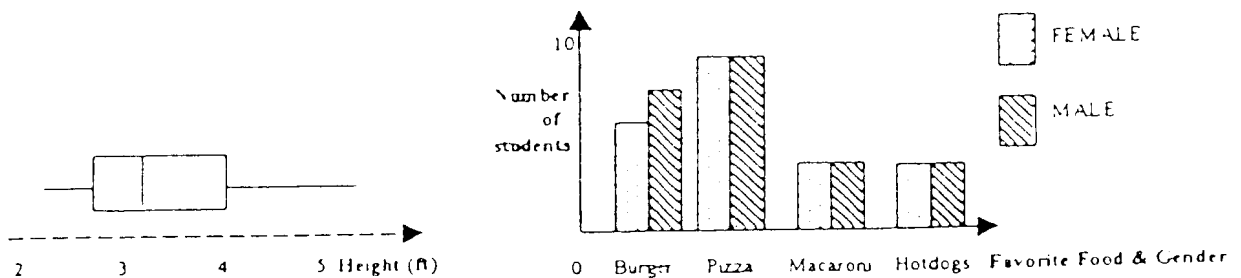
D) A scatter plot is the best way to depict unpaired data.

E) Experimental probability always gives the same results as theoretical probability.

VI A) A survey of 100 college faculty who dine out regularly found that 32 eat pizza, 30 eat burgers, 15 eat salads, 6 eat pizza and salad, 1 eats pizza and burgers, 5 eat salad and burgers, and 1 eats all three. (Hint: Draw a diagram to help you.)

- i) How many of the faculty members surveyed do not eat any of these three selections?
- ii) How many faculty members just eat salad (of the three choices given)?
- iii) What is the probability that a burger-eating faculty will also eat pizza or eat salad?

B) The box-and-whisker plot and histogram below represent sample data taken from a study with 50 elementary students. Answer the questions based on the information given. If the information needed cannot be obtained from the graphs, SAY SO.

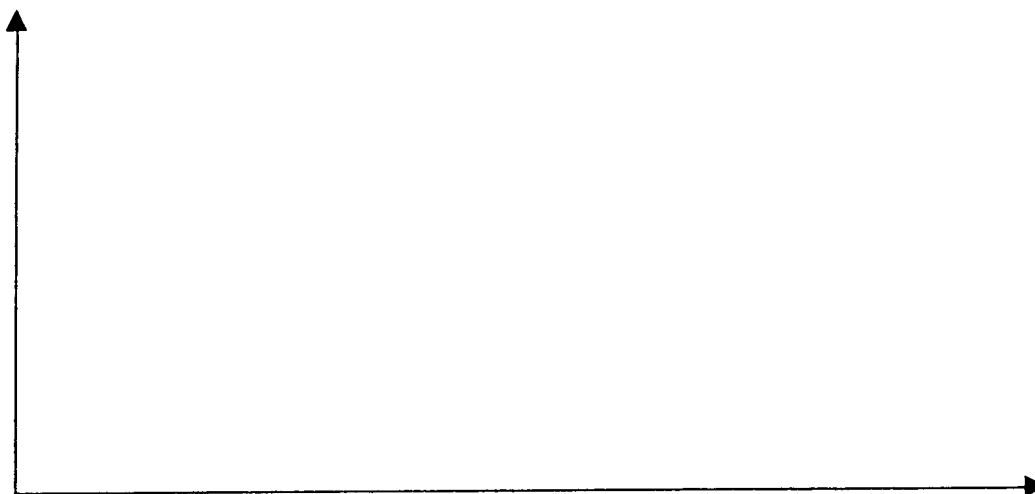


- i) What is the mean height of the students in the sample?
- ii) Find the number of students in the sample who are at least 4 ft tall.
- iii) How many students in the sample are female?
- iv) What is the probability that a male student is over 4 ft tall?
- v) What percentage of sample students say their favorite lunch is pizza?

- VII A) Draw a graph representing the height of water in a bathtub as it is filled, used, and emptied as follows:

At the beginning, the tub is empty. The drain is plugged and the tap is opened gradually to its fullest extent. The tap is closed after the tub is half filled. After some time has passed, someone gets into the tub. After bathing, the person gets out of the tub, and then pulls the plug and allows the water to drain.

Note: Label the axes appropriately and identify points on your graph with what happens at different stages.



- B) Pappy Snappy, a math enthusiast, has determined that by iterating the expression x^3 he always gets a larger number. Is this the whole story? Give the “whole story” and justify your response.

VIII A) Think of a problem in a context appropriate for elementary school children that illustrates the operation of division as repeated subtraction.

i) Pose the problem as a question.

ii) Solve the problem illustrating the use of repeated subtraction.

B) Think of a real-life situation (choose a context different to those appearing in this exam) where there is a functional relationship between an independent and dependent variable.

i) Describe the situation in words, graph, table OR equation.

ii) Identify the independent variable and explain why the relationship is a function.